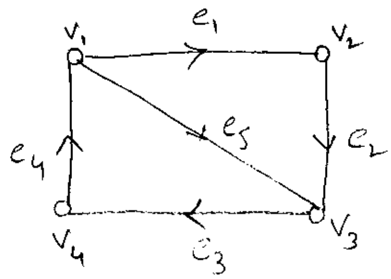
UNIT-VIII Elementary Graph Theory.

(1) Find the Adjacency matrix to represent the following directed graph. Also find Incidence matrix & degree of vertices.



Sol:- We order the vertices as v_1, v_2, v_3, v_4.

Since there are 4 vertices, the adjacency matrix 'A' is 4x4 matrix.

$$\therefore \text{Adjacency matrix } A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\text{Incidence matrix } A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

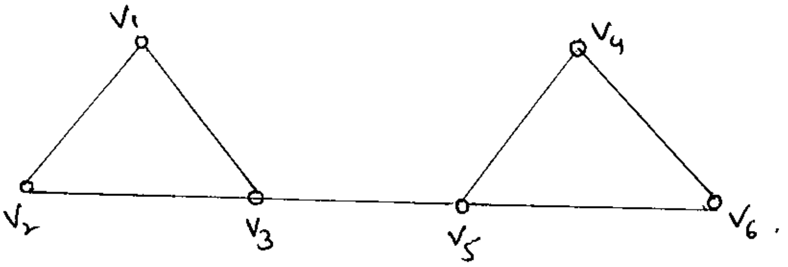
$deg(v_1) =$ Indegree + outdegree $= 1 + 2 = 3$.

$deg(v_2) = 1 + 1 = 2$

$deg(v_3) = 2 + 1 = 3$.

$deg(v_4) = 1 + 1 = 2$.

(2) Find ~~the~~ all spanning trees for the following graph.

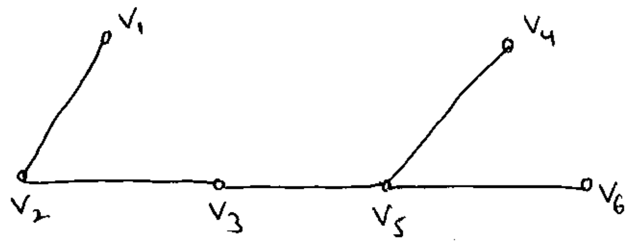


Sol:- The given graph has 6 vertices. So each spanning tree must have $6-1=5$ edges.

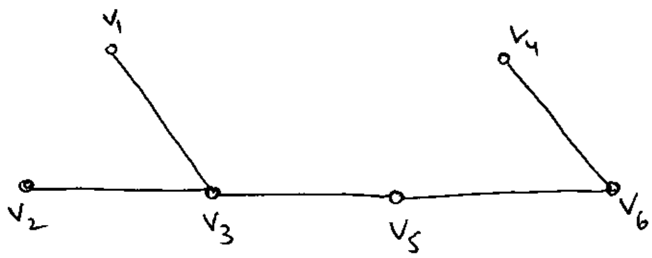
The given graph has 7 edges. So 2 edges have to be deleted.

The graph has 2 cycles. Removing the edges which are forming cycles yields spanning trees. Hence all the spanning trees of given graph are

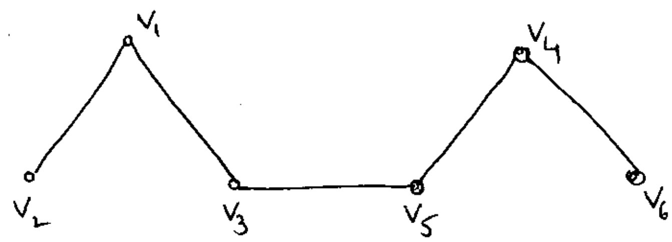
(i)



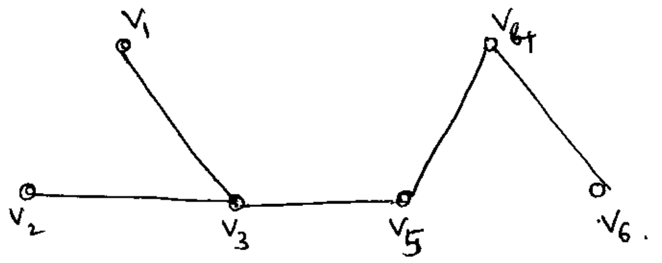
(ii)



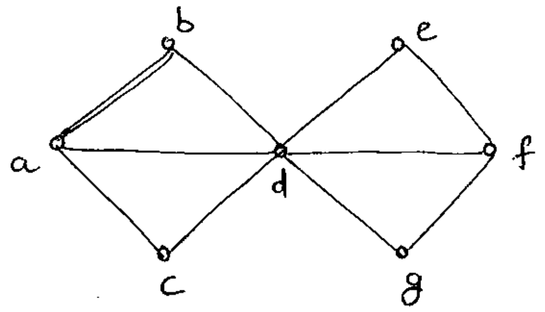
(iii)



(iv)



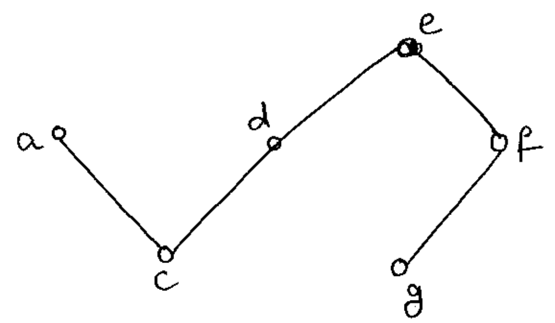
(3) Find the Spanning tree for the following graph by applying DFS Algorithm.



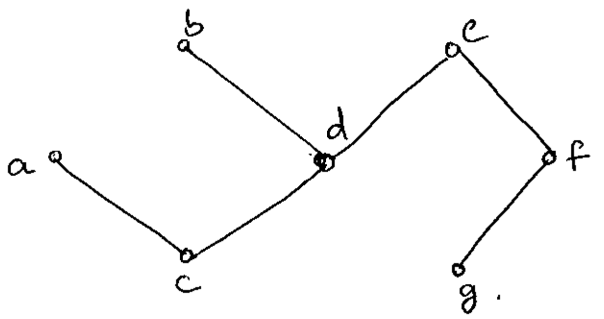
Sol:- As in DFS algorithm, arbitrarily choose a vertex of the graph as a root. ~~cons~~' let 'a' as the root of the graph. Construct a path starting at this vertex by successively adding edges as long as possible, each edge is incident with vertices not already in the path.

Then start with 'a' and it produces the path

a – c – d – e – f – g.



Now back to d' from d-b.

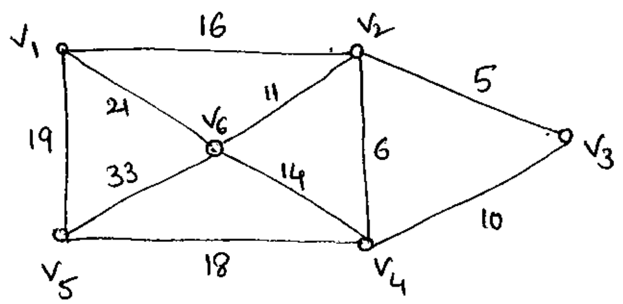


(the paths b-a, g-d forms the cycles.)

This path goes through all vertices of the graph.

∴ This is the required Spanning tree.

(4) Use kruskal's algorithm to find a minimal spanning tree for the following weighed graph.



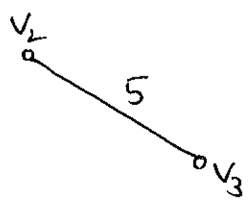
Sol:- The given graph has '6' vertices.

$V_1, V_2, V_3, V_4, V_5, V_6$.

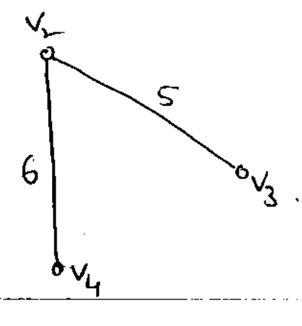
The following table gives the edges with their corresponding weights.

Edge	Weight
V_2-V_3	5
V_2-V_4	6
V_4-V_3	10
V_2-V_6	11
V_4-V_6	14
~~V_4-V_5~~	~~18~~
V_2-V_1	16
V_4-V_5	18
V_5-V_1	19
V_6-V_1	21
V_6-V_5	33

Step (i):- choose the edge V_2-V_3 as it is minimum weight.



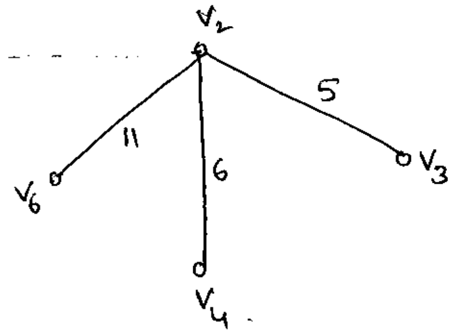
(ii) Add next edge with minimum weight i.e., V_2-V_4.



(iii) Add the edge $V_4 - V_3$.

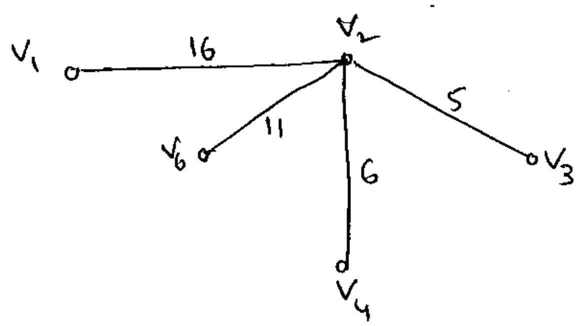
if we add this edge it forms a cycle. so Reject this.

(iv) Add the edge $V_2 - V_6$

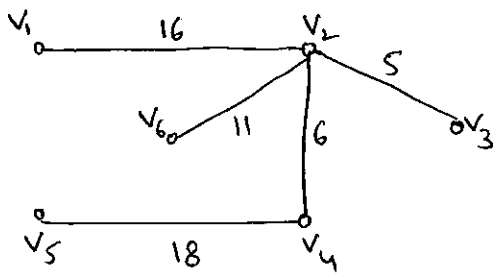


(v) Reject $V_4 - V_6$ since it forms cycle.

(vi) Add edge $V_2 - V_1$



(vii) Add edge $V_4 - V_5$.



(viii) Reject the edges $V_5 - V_1$, $V_6 - V_1$, $V_6 - V_5$ since these all edges form cycles.

Therefore all the vertices of given graph are covered.

So we stop the algorithm.

∴ The above graph produces the minimal spanning tree for the given graph.

The minimal cost for construction of this tree =

$$5 + 6 + 11 + 16 + 18 = 56.$$