UNIT - 8

Graph theory and Applications

1. Suppose that we know the order and size of a non-directed graph G. Is it possible to determine the degrees of the vertices of G? Explain.

A) No, as we know order and size of a nondirected graph G it is not possible to find the degrees of the vertices of G.

Here order means number of vertices and size means sum of their degree.

Let us see this with one Example.

Suppose that a non-directed graph having order 8 and size of a graph is 14.

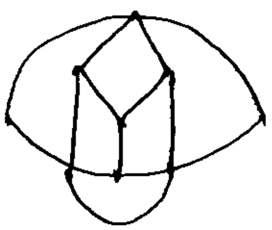
then number of edges = 2 × size

= 2 × 14 = 28

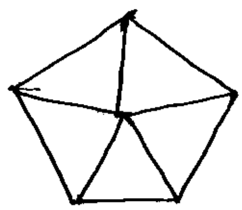
from above order and size it is not possible to find the degrees of vertices of G.

2. Find the chromatic number of the following graphs

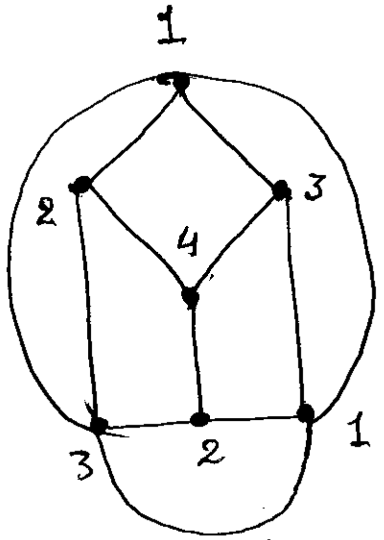
a)



b)



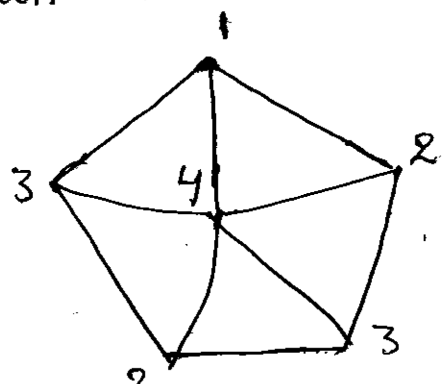
A) a) The proper colouring of the graph is given below.



The graph is properly coloured with 4-colours.

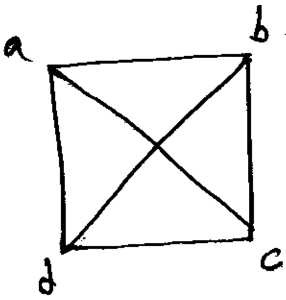
$\therefore$ The chromatic number for the first graph is 4.

b) The given graph is wheel graph with 6-vertices. Since the number of vertices is even, the chromatic number for the second graph is 4 i.e. $\chi(G) = 4$. The graph can be coloured as shown below.



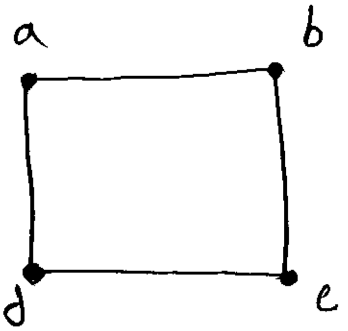
$\therefore \chi(G) = 4$

(3) check whether Hamiltonian cycle and Hamiltonian path are there or not?



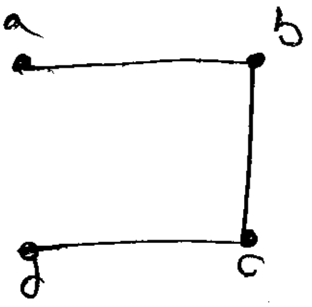
A) Hamiltonian Cycle :-

There are no vertices of degree two in the given graph. Hence start from largest degree vertex which is 3. Consider any vertex in the graph. Let it be 'a', we have one edge at 'a'. delete any arbitrary edge. Let us delete {a, c} Now select {a, b} and {a, d} at vertex 'a' and {c, b} and {c, d} at c. At 'b' already two edges are selected. therefore delete {b, d}. Thus the degrees of all vertices has becomes two. Hence there is Hamiltonian cycle and the cycle is



a – b – c – d – a

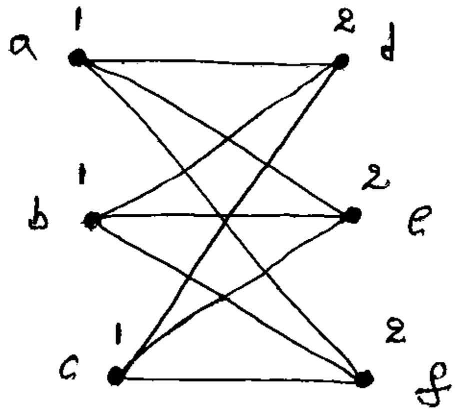
Hamiltonian path: Since the graph has Hamiltonian cycle it also contain Hamiltonian path. delete any one Edge from Hamiltonian cycle, we get Hamiltonian path the Hamiltonian path is



a-b-c-d

(4) Find the chromatic number of a bipartite graph $K_{3,3}$

(A) A complete bipartite graph $K_{3,3}$



$\therefore \chi(K_{3,3}) = 2$

$\therefore$ Chromatic number of $K_{3,3}$ is 2.

(5) List out the rules to find chromatic number of a given graph?

1. The chromatic number of a isolated vertex is one (Because no two vertices of such a graph are adjacent and therefore we can assign the same colour to all vertices).

2. A graph with one or more edges is at least 2-chromatic z-chromatic

3. The chromatic number of a path P_n, $(n \geq 2)$ is two.

4. if graph G contains a graph G_1 as a subgraph then $\chi(G) \leq \chi(G_1)$.

5. if G is graph of 'n' vertices, then $\chi(G) \leq n$.

6. $\chi(K_n) = n$ for all $n \geq 1$

7. if a graph G contains K_n as a subgraph then $\chi(G) \geq n$.

8. The chromatic number of a whell graph is 3 if it has an odd number of vertices and 2 if it has an even vertices

i.e., $\chi(n) = 3$ if n is odd

$\chi(C_n) = 2$ if n is even

9) The chromatic number of a complete graph K_n is 'n'.

10) chromatic number of trivial tree (i.e, one vertex) is one.

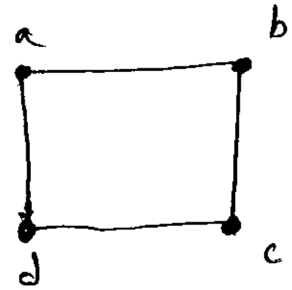
11) chromatic number of non-trivial tree is 2.

12) chromatic number of $K_{m,n}$ is 2 if $m = n$ (or) $m \neq n$

13) If G is a graph consisting of simply a circuit with $n \geq 3$ then G is 2-chromatic if n is even and 3-chromatic if n is odd.

14) $\chi(C_n) = 3$ if n is even and $\chi(W_n) = 4$ if n is odd.

(6) check whether Hamiltonian Cycle and Hamiltonian path are there or not?

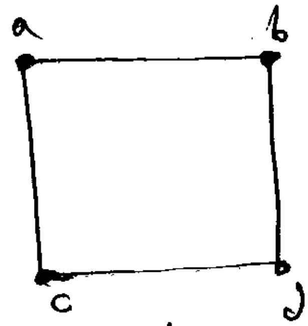


A) <u>Hamiltonian cycle</u>:- 1. There are no vertices of degree two in the given graph. Hence start from largest degree vertex which is 3.

2. Consider any vertex in the graph. Let it be 'a'. we have one edge at 'a' delete any arbitrary edge.

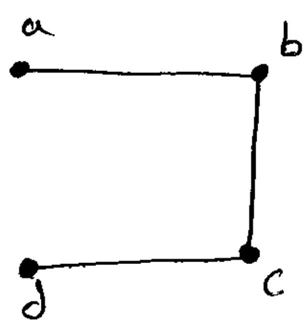
3. Let us delete {a, c}, now select {a, b} and {a, d} at vertex 'a' and {c, d} and {c, b} at c. At 'b' already two edges are selected.

4. Therefore delete {b, d}. Thus the degree at all vertices has become two. Hence there is Hamiltonian cycle and the cycle is



a - b - c - d - a

Hamiltonian path: Since the graph has Hamiltonian cycle it also contain Hamiltonian path. Delete any one edge from Hamiltonian cycle, we get Hamiltonian path the Hamiltonian path is

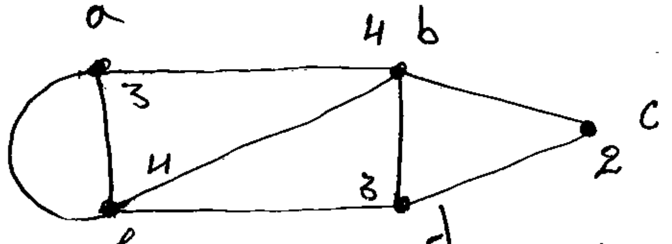


a - b - c - d

(9) Define Euler path and Euler Circuit with Example.

Euler path : 1. An Euler path is defined as all the edges must be traversed Exactly once and all the vertices must be traversed at least once.
2. the starting vertex need not be same as ending vertex. To get Euler path the degree of all vertices must be even except two vertices whose degree is odd. To get Euler path we have to start from one of the odd degree vertices and path terminates at other odd degree vertex.

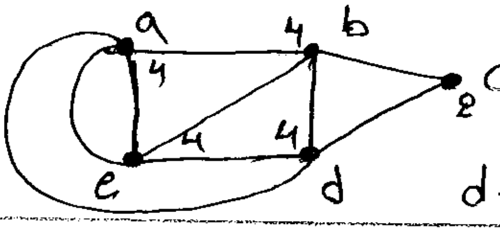
EX :-



Euler path d – c – b – a – e – d – b – e – a

Euler Circuit :- An Euler circuit is defined as start from a vertex travel all the edges exactly once. Travel all vertices atleast once and come back to starting vertex. This is Euler Circuit starting vertex is equal to ending vertex where as in Euler path starting vertex is not equal to ending vertex. To get Euler circuit degrees of all vertices must be even i.e. no vertices of odd degree should exist in the graph.

EX :-



Euler Circuit is d – c – b – a – e – d – b – e – a – d